Technische Universiteit
Eindhoven
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Lattice-based cryptography

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PQC Executive School
(July 1, 2019)

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Lattice-based signatures

Lattice algorithms

Summary

References

Outline

Lattices

What is a lattice?

Lattices

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Lattices

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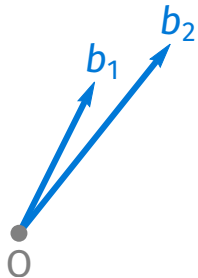
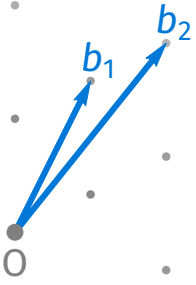


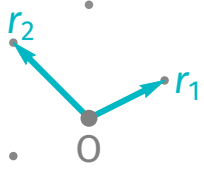
Lattices

What is a lattice?



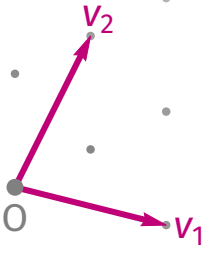
Lattices

Basis for the same lattice



Lattices

Basis for a sublattice



Lattices

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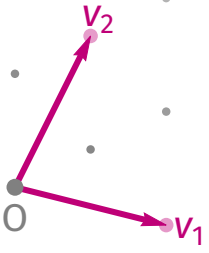

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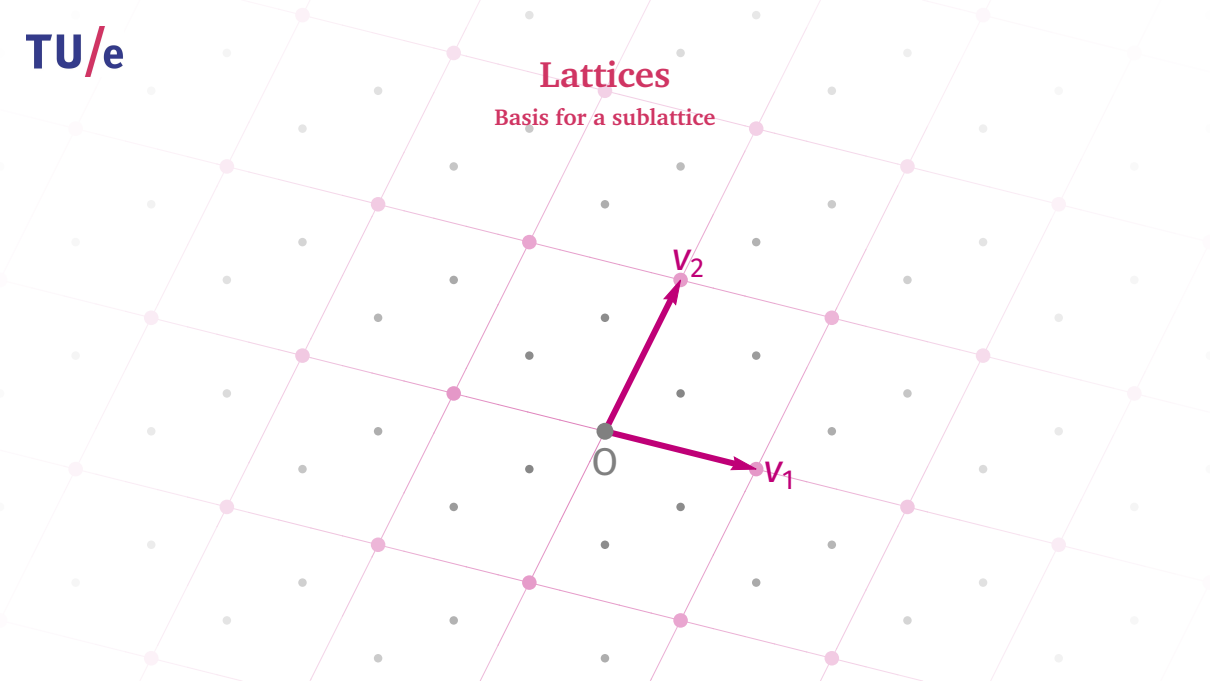
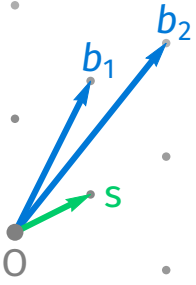


Lattices

Shortest Vector Problem (SVP)



Lattices

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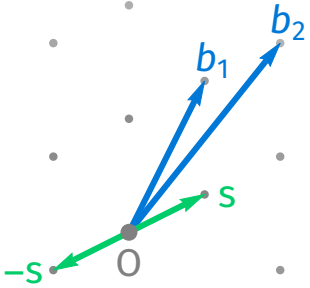

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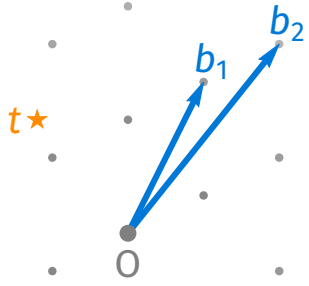

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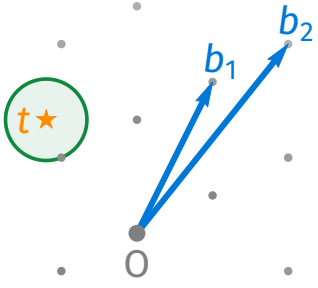

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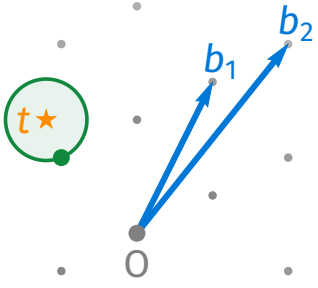

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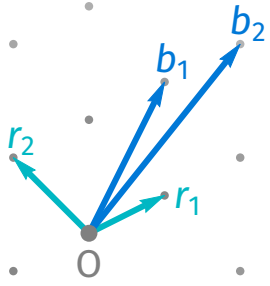
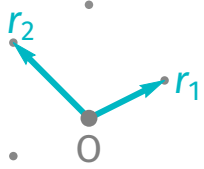
Lattices

Lattice basis reduction



Lattices

Finding closest vectors (good basis)



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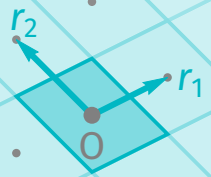

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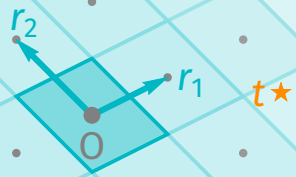

Lattices

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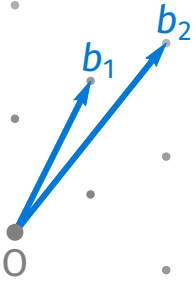
Finding closest vectors (bad basis)



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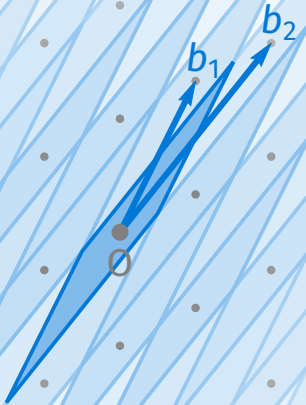

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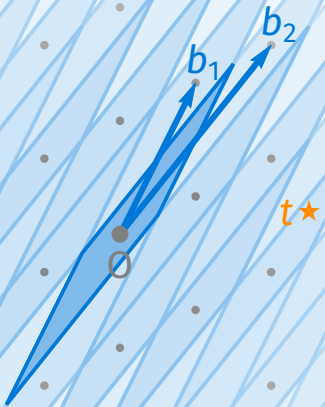

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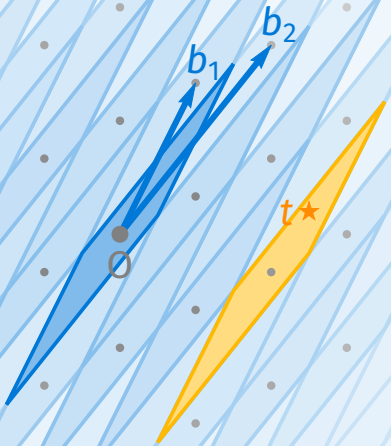

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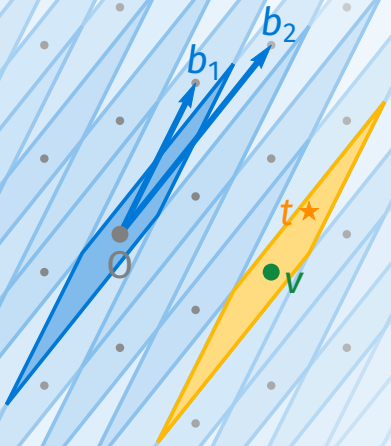

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Lattices

Lattice problems

Easy lattice problems:

- Given a good basis, finding a bad basis
- Given a good basis, finding short lattice vectors
- Given a good basis, finding close lattice vectors
- Given any basis, verifying membership of the lattice

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Lattices

Lattice problems

Easy lattice problems:

- Given a good basis, finding a bad basis
- Given a good basis, finding short lattice vectors
- Given a good basis, finding close lattice vectors
- Given any basis, verifying membership of the lattice

Hard lattice problems:

- Given a bad basis, finding a good basis
- Given a bad basis, finding short lattice vectors
- Given a bad basis, finding close lattice vectors

Outline

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Lattice-based signatures

Overview [GGH97]

Private key:

$\boldsymbol{R} = \{\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n\}$

Public key:

$\boldsymbol{B} = \{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n\}$

Signing $\boldsymbol{m}$:

$\boldsymbol{c} = H(\boldsymbol{m})$

$\boldsymbol{s} = \boldsymbol{R}\lfloor \boldsymbol{R}^{-1}\boldsymbol{c} \rceil$

Verifying $\boldsymbol{s}$:

$\boldsymbol{s} \in \mathcal{L}$

$\|\boldsymbol{s} - H(\boldsymbol{m})\|$ small

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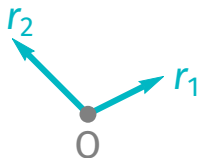

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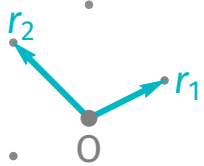

Lattice-based signatures

Public key

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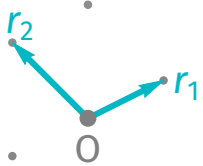

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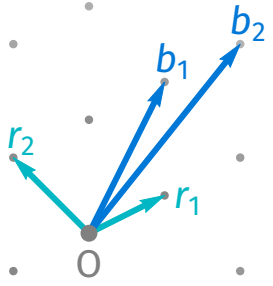

Lattice-based signatures

Signing a message

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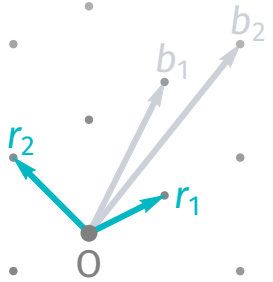

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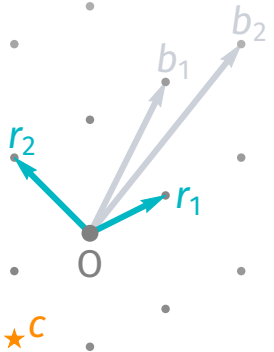

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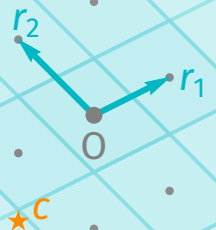

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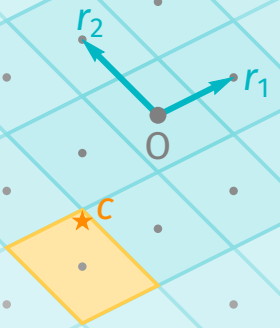

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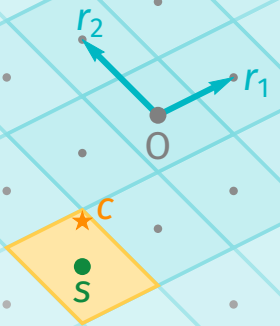

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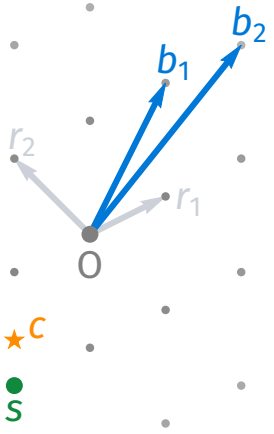

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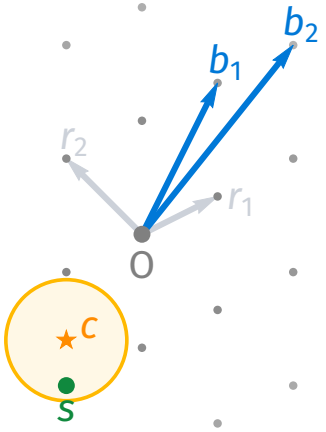

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Lattice-based signatures

Overview

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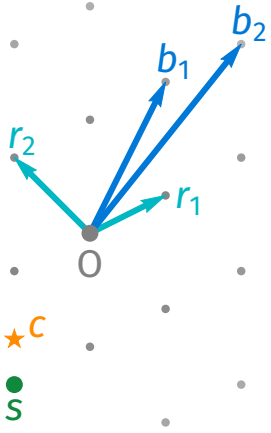

Lattice-based signatures

Attacking the scheme [NR06]

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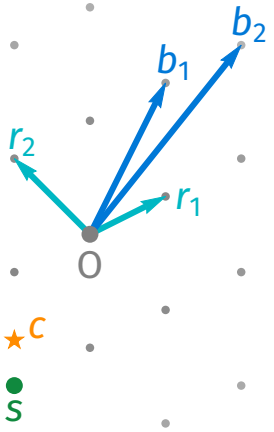

Obtain error vectors:

$\boldsymbol{s} = \mathrm{Sign}(\boldsymbol{m})$

$\boldsymbol{c} = H(\boldsymbol{m})$

$\boldsymbol{e} = \boldsymbol{c} - \boldsymbol{s}$

Leaks $\boldsymbol{R} = \{\boldsymbol{r}_1, \ldots, \boldsymbol{r}_d\}$:

$\boldsymbol{e} \in \mathcal{D}(\boldsymbol{R})$

Can recover $\boldsymbol{R}$ via gradient descent

Lattice-based signatures

Obtain a signature

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Signing m:

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$s = R\lfloor R^{-1}c \rceil$

Verifying s:

$s \in \mathcal{L}$

$\|s - H(m)\|$ small

Obtain error vectors:

$s = \text{Sign}(m)$

$c = H(m)$

$e = c - s$

Leaks $R = \{r_1, \ldots, r_d\}$:

$e \in \mathcal{D}(R)$

Can recover R via gradient descent

Lattice-based signatures

Compute the error vector

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$c = H(m)$

$e = c - s$

Leaks $R = \{r_1, \ldots, r_d\}$:

$e \in \mathcal{D}(R)$

Can recover R via gradient descent

Lattice-based signatures

Store the error vector

Private key:

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Public key:

$B = \{b_1, \ldots, b_n\}$

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Verifying s:

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Obtain error vectors:

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$e = c - s$

Leaks $R = \{r_1, \ldots, r_d\}$:

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Can recover R via gradient descent

TU/e

Lattice-based signatures

Obtain a signature

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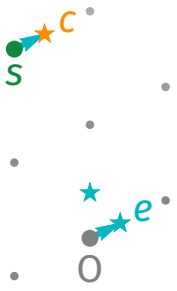

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$s = \text{Sign}(m)$

$c = H(m)$

$e = c - s$

Leaks $R = \{r_1, \ldots, r_d\}$:

$e \in \mathcal{D}(R)$

Can recover R via gradient descent

Lattice-based signatures

Store the error vector

Private key:

$R = \{r_1, \ldots, r_n\}$

Public key:

$B = \{b_1, \ldots, b_n\}$

Signing m:

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Verifying s:

$s \in \mathcal{L}$

$\|s - H(m)\|$ small

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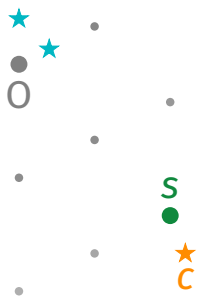

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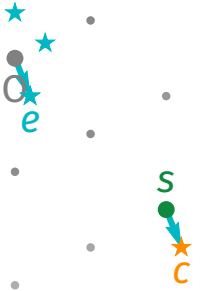

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TU/e

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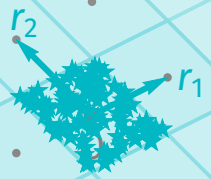

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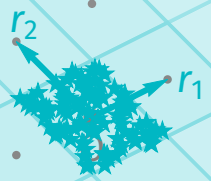

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Lattice-based signatures

Overview

Private key:

$\boldsymbol{R} = \{\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n\}$

Public key:

$\boldsymbol{B} = \{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n\}$

Signing $\boldsymbol{m}$:

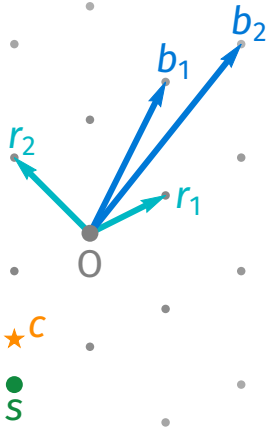
$\boldsymbol{c} = H(\boldsymbol{m})$

$\boldsymbol{s} = \boldsymbol{R}\lfloor \boldsymbol{R}^{-1}\boldsymbol{c} \rceil$

Verifying $\boldsymbol{s}$:

$\boldsymbol{s} \in \mathcal{L}$

$\|\boldsymbol{s} - H(\boldsymbol{m})\|$ small



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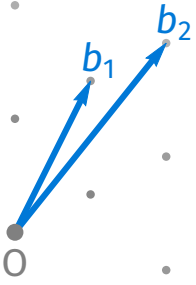
$\boldsymbol{e} \in \mathcal{D}(\boldsymbol{R})$

Can recover $\boldsymbol{R}$ via gradient descent

Outline

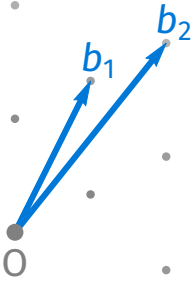
Lattice algorithms

Lattice enumeration [FP85]



Lattice algorithms

Determine possible coefficients of b_2



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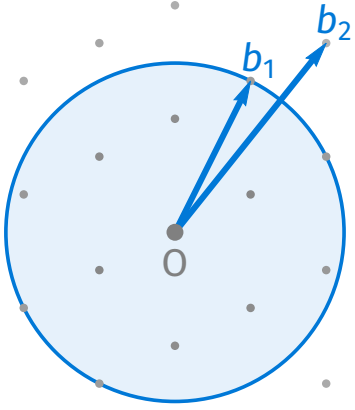

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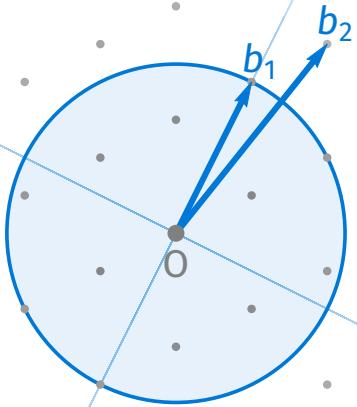

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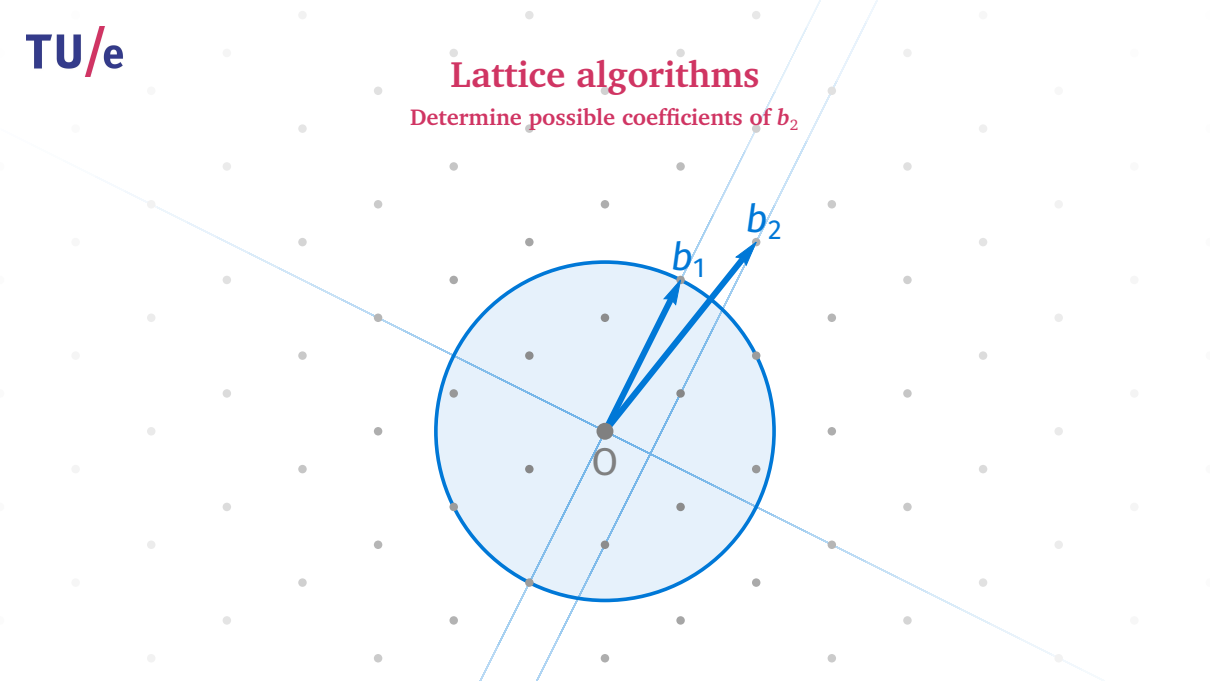

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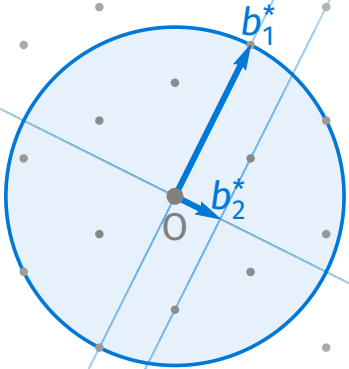

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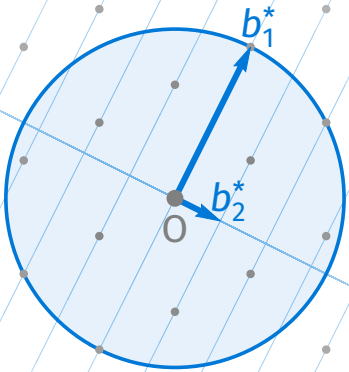
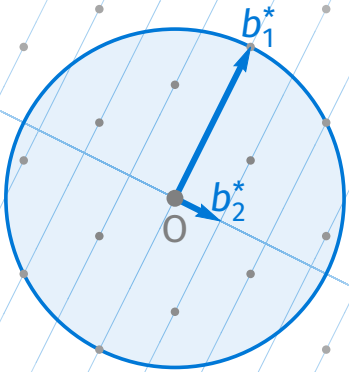


Lattice algorithms

Find short vectors for each coefficient of b_2



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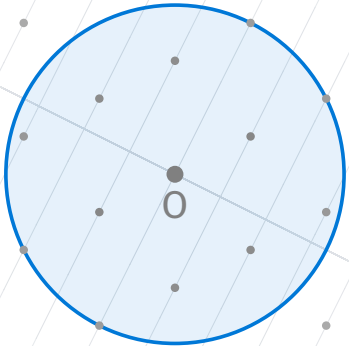

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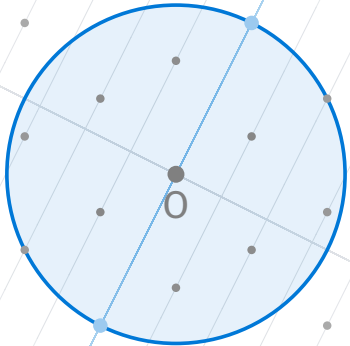

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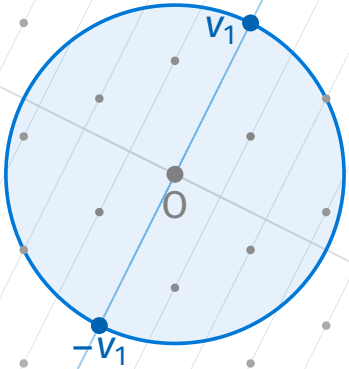

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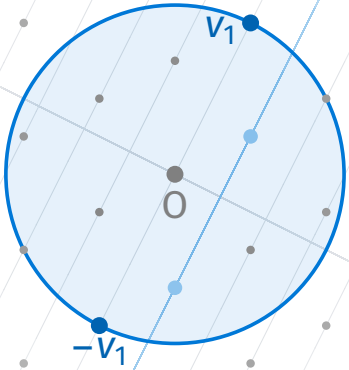

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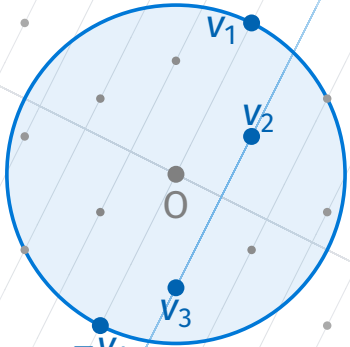

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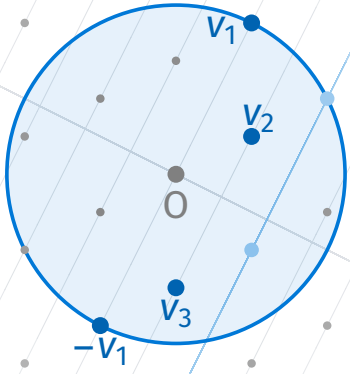

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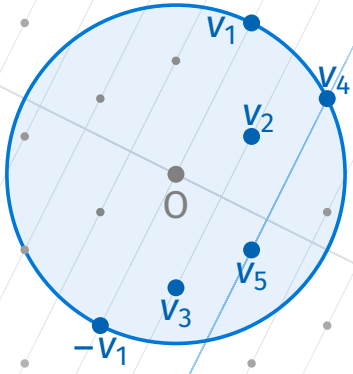

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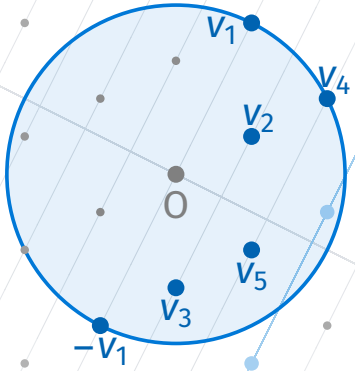

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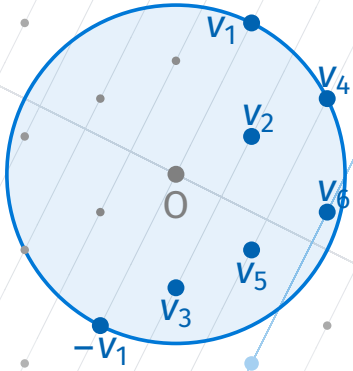

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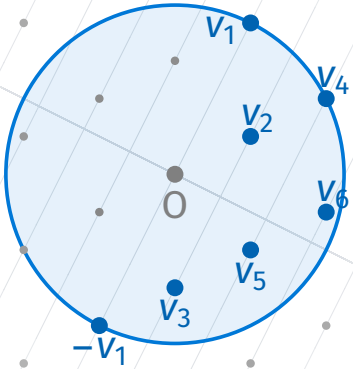

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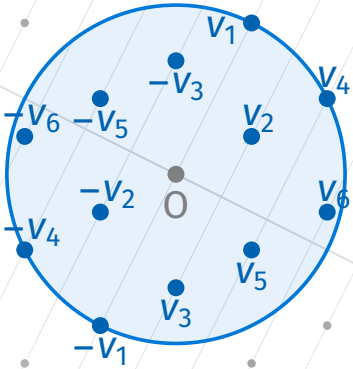
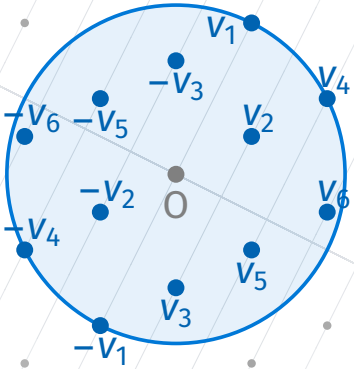


Lattice algorithms

Find a shortest vector among all found vectors



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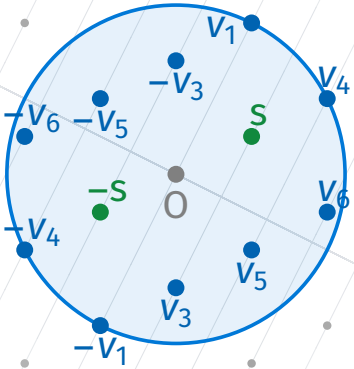

TU/e

Lattice algorithms

Lattice enumeration vs. lattice sieving

Lattice enumeration

- Brute-force all combinations of the basis vectors [FP85]
- Each coordinate $2^{O(n)}$ options $\implies$ $2^{O(n^2)}$ time complexity
- Balance preprocessing time with search time: $2^{O(n \log n)}$ time [Kan83]
- Requires almost no memory

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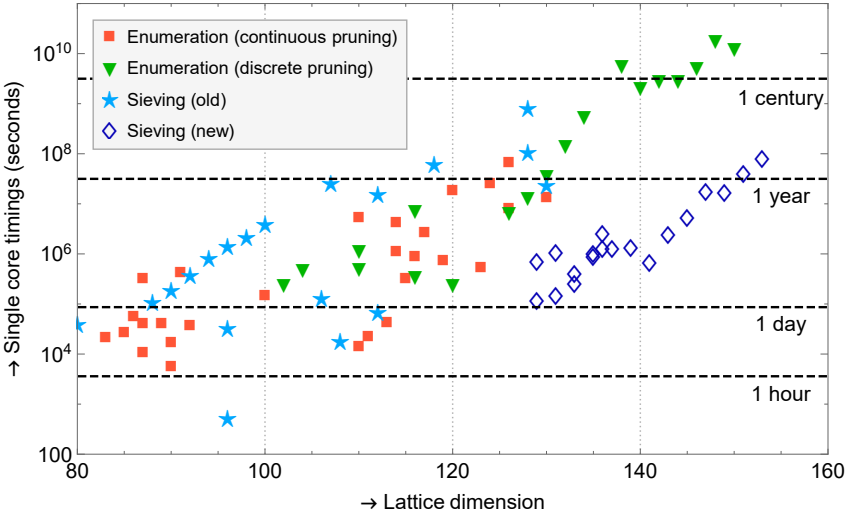
Lattice sieving

- Combine vectors in a huge list to find shorter vectors [AKS01]
- Generally requires $2^{O(n)}$ time and $2^{O(n)}$ space
- Current best asymptotics: $2^{0.292n}$ time and space [BDGL16]

Lattice algorithms

Practice [SVP]



TU/e

Lattice algorithms

Concrete estimates

Most common hardness estimates:

- Cost of finding nice basis (n) $\geq$ Cost of sieving/enumeration (β)
- Ignore space complexity, ignore $o(\beta)$ in time complexity
- Classical sieving: $2^{0.292\beta}$ time [BDGL16]
- Quantum sieving: $2^{0.265\beta}$ time [Laa16]
- "Paranoid bound": $2^{0.208\beta}$ time

Outline

TU/e

Summary

Lattice-based cryptography

- Main principle: hard to find good bases from bad bases
  - Private key: Good lattice basis
  - Public key: Bad lattice basis
- Signatures: Decode with good basis to nearby lattice point

Lattice-based cryptanalysis

- Two flavors, trading time for memory
  - Enumeration: Low memory, slower in high dimensions
  - Sieving: High memory, faster in practice
- Security estimates: solving SVP/CVP takes $\geq 2^{0.292n}$ time

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Next talk: Key exchange, encryption, side-channel, attacks and more!

TU/e

References (clickable)

Lattice-based cryptography

- General: [MG02], [MR09], [Mic16], [Pei16], [NIST]
- Signatures: [GGH97], [NR06], [DN12]
- NTRU: [HPS98], [HHPW10], [SS11], [BCLV17]
- LWE: [Reg05], [Reg10], [BCD+16]
- Ring-LWE: [SSTX09], [LPR10], [LPR13]
- (Ring-)LWR: [BPR13], [AKPW13], [AKRV17], [BBF+17]

Lattice-based cryptanalysis

- General: [MG02], [MR09], [HPS11], [Mic16], [ACD+18], [SVP]
- Enumeration: [FP85], [Kan83], [GNR10], [AN17], [ANS18]
- Sieving: [AKS01], [LMP15], [BDGL16], [Laa16], [Duc18], [ADH+19]
- Basis reduction: [LLL82], [SE94], [CN11]